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CALCULATION OF THE EQUILIBRIUM GEOMETRY OF A CABLE BEING LAID WITH NON-ZERO BOTTOM TENSION

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Environmental Sciences Department

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A method is derived for calculating the equilibrium geometry and tension of a cable being laid with non-zero bottom tension in the presence of a cross current. A computer program is presented which performs the calculations.

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INTRODUCTION

NORMAL CABLE LAYING

Normally the people who are concerned about laying cables in the ocean are involved in laying undersea telephone cables.[1] They have available elaborate methods for measuring and adjusting ship speed, cable payout rate, and cable tension. Since it is thought that most cable breaks occur when the cable has been suspended above the bottom due to obstructions or abrupt changes in bottom slope, an attempt is made to lay the cable with a percentage of slack. That is, more cable is laid out than distance covered. In this way it is hoped that small bottom irregularities will be filled in. As large scale changes in bottom depth occur, and the length of bottom which must be covered is greater than the horizontal distance which the ship moves, a large percentage of slack is laid to cover the increased bottom distance.

Cables have a characteristic transverse sinking speed, U_S, at which if a length of cable were falling horizontally through the water, the cable's net weight would be balanced by the drag force. This sinking speed and the velocity of the cable ship, V_S, determine the angle of cable as it is being laid, figure 1. Thus during normal cable laying operations, the cable forms a straight line from the ship to the bottom.

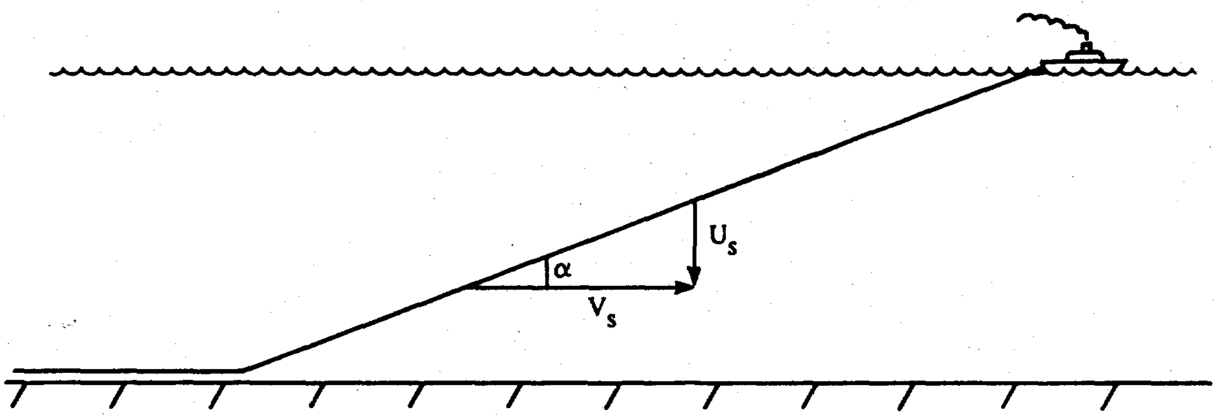

Figure 1. Cable geometry with zero bottom tension.

Where the cable touches the bottom, cable tension is zero. At the ship, cable tension is equal to the weight of a vertically hanging cable the length of which is equal to the water depth.

If a cross current with velocity U_c is present, the track of the cable is displaced from the track of the ship as shown in figure 2. However, the geometry of the cable is still a straight line, and the cable tension at the ship is still the same as in the case with no current.

[1] Roden, C. E., Submarine Cable Mechanics and Recommended Laying Procedures, Bell Telephone Laboratories, December, 1964.

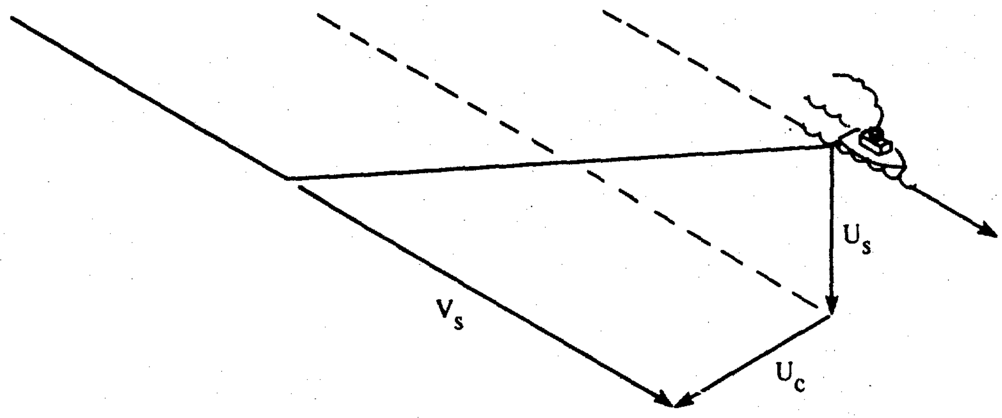

Figure 2. Cable geometry with zero bottom tension and a cross current.

When trying to visualize what is happening during cable laying, it is important to avoid the fallacy of thinking of the cable as being suspended in a catenary between the ship and the bottom. Actually in normal cable laying, most of the cable weight is supported by the drag created as the cable falls through the water.

LAYING WITH BOTTOM TENSION

During some cable lays, the precise measurements and adjustments of ship speed and cable payout speed normally used by cable layers are not available. Instead, the cable payout tension is preset in the cable deployment system. In this case, cable tension at the bottom is no longer equal to zero, and the cable is no longer straight. Instead it is curved near the bottom as in figure 3.

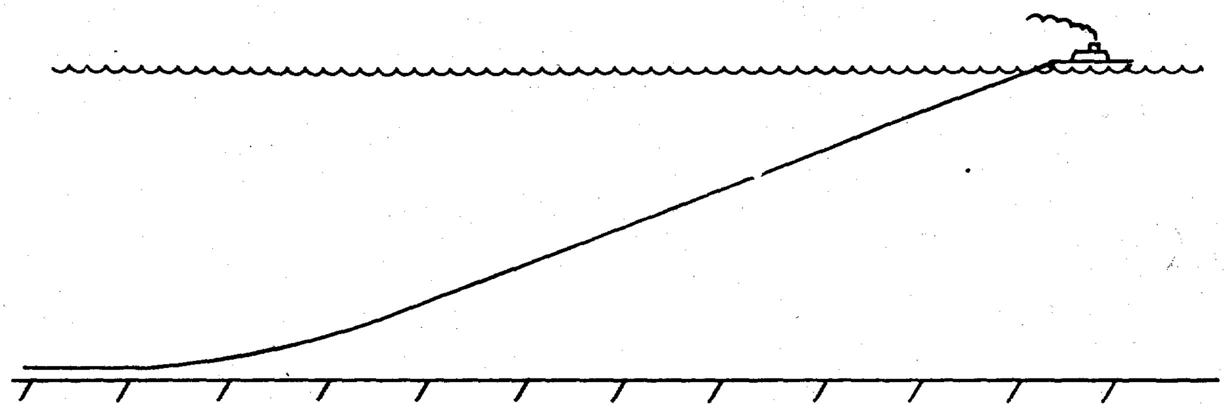

Figure 3. Cable geometry with non-zero bottom tension.

This report presents a calculation method and a computer program which ~~can~~ be used to determine cable geometry and tension during cable laying with non-zero bottom tension and with a cross current. The current can be set equal to zero if the simple two dimensional configuration is desired.

DERIVATION OF EQUATIONS

BASIC PRINCIPLES

There are three basic principles which are fundamental to the derivation of these equations.

First, since the cable is flexible, it can only exert a force which lies along its length. Referring to figure 4, the cable tension vector always lies along the cable. This can be expressed as:

(1) $$\frac{dx}{dL} = \frac{T_x}{T}$$

(2) $$\frac{dy}{dL} = \frac{T_y}{T}$$

(3) $$\frac{dz}{dL} = \frac{T_z}{T}$$

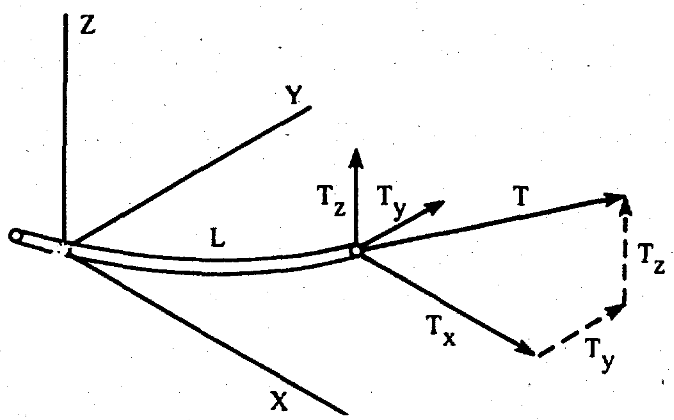

Figure 4. Cable tension vector.

Second, with fixed bottom conditions, the cable configuration below a certain point on the cable is independent of how much cable is above that point. In figure 5, if the bottom tensions and ship speeds are the same, the geometry of the cable below point P is identical to the geometry of the cable below point P′. Only the tensions at the two ships will be different.

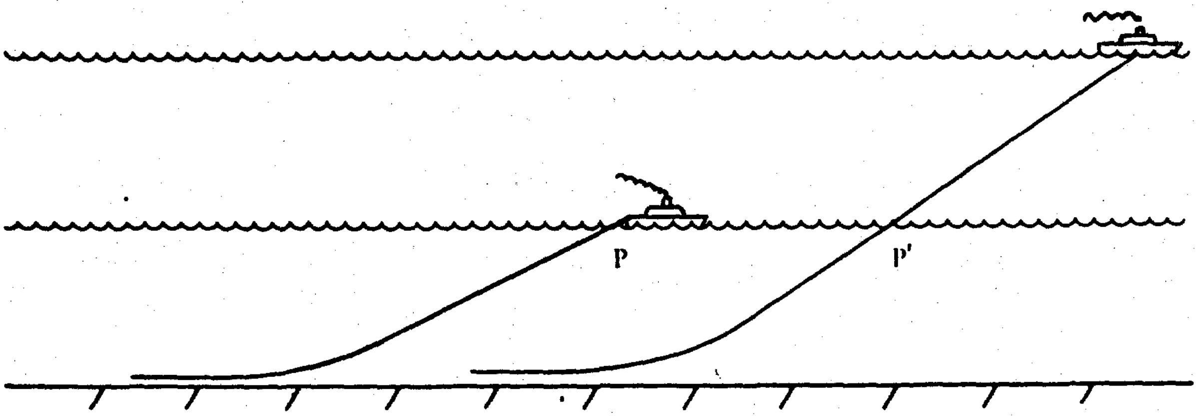

Figure 5. Cable geometry with two different bottom depths.

Third, in steady state laying, cable velocity through the water is directly related to cable geometry. In figure 6, points P_1 and P_2 represent two points on the cable at time, t. At some later time, $t + \Delta t$, the whole cable geometry will have moved to the right a distance equal to $\Delta L = (\Delta t)(V_s)$. Point P_2 will have moved to P_3, which corresponds to the same position on the new cable configuration as point P_1 held on the cable configuration at time t. The vertical component of the cable velocity through the water is:

$$V_z = \frac{\Delta z}{\Delta t}$$

$$= \frac{-(\Delta L)\left(\frac{dz}{dL}\right)}{\Delta L / V_s}$$

(4) $$V_z = -V_s \frac{dz}{dL}$$

Or using equation 3:

(5) $$V_z = -V_s \frac{T_z}{T}$$

In a similar manner:

(6) $$V_x = V_s \left(1 - \frac{T_x}{T}\right)$$

(7) $$V_y = U_c - V_s \frac{T_y}{T}$$

where U_c is the velocity of the cross current.

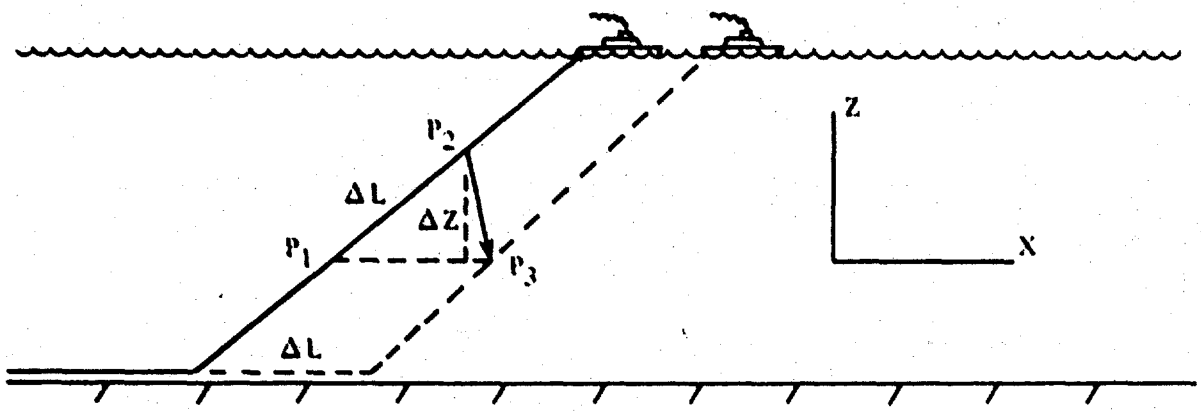

Figure 6. Cable geometry at two different times.

TOUCHDOWN POINT

Conditions are known for the point at which the cable touches the bottom. It is shown in reference 1 and will be shown in later examples that the tension in the cable at the point where it touches the bottom is equal to the tension at the cable ship minus the net weight of a vertically hanging cable. Thus the tension at the bottom, T_0, is known.

Since the cable is being laid in a straight line in the x direction:

(8) $$T_y = 0$$

Since there is a net bottom tension, the cable will be horizontal where it touches the bottom. Therefore:

(9) $$T_z = 0$$

Since T_y and T_z are zero:

(10) $$T_x = T_o$$

CABLE SEGMENTS

Now consider adding a short cable segment. The geometry gets rather complicated and is most easily handled using vector notation (figure 7).

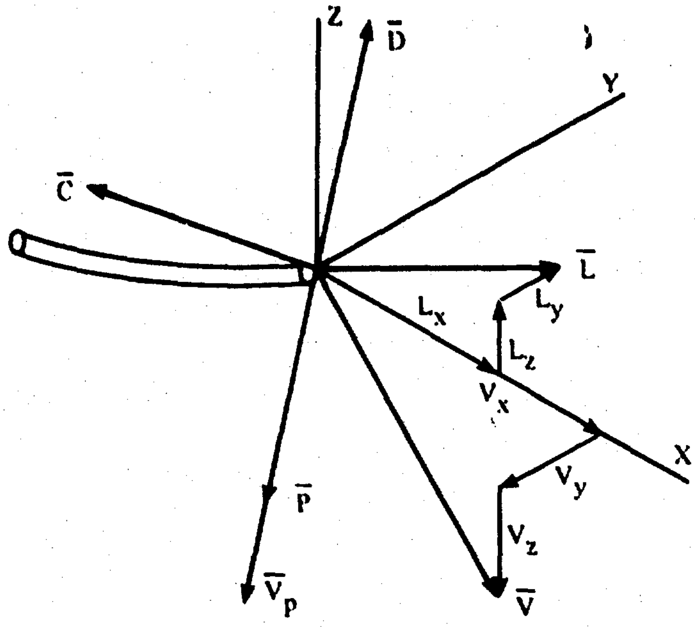

Figure 7. Vector geometry at a point on the cable.

Let $\bar{L}$ be the unit vector which lies along the cable.

(11) $$\bar{L} = L_x \bar{i} + L_y \bar{j} + L_z \bar{k}$$

From equations 1-3:

(12) $$L_x = \frac{T_x}{T}$$

(13) $$L_y = \frac{T_y}{T}$$

(14) $$L_z = \frac{T_z}{T}$$

Let $\bar{V}$ be the velocity of the cable with respect to the water at any point along the cable.

(15) $$\bar{V} = V_x \bar{i} + V_y \bar{j} + V_z \bar{k}$$

where the terms V_x, V_y, V_z are found from equations 5-7.

Define $\bar{C}$ as a unit vector perpendicular to $\bar{L}$ and $\bar{V}$.

(16) $$\bar{C} = \frac{\bar{V} \times \bar{L}}{|\bar{V} \times \bar{L}|}$$

Define $\bar{P}$ as the third unit vector making up a right hand coordinate system. It is

perpendicular to $\overline{L}$ and $\overline{C}$ and in the plane of $\overline{L}$ and $\overline{V}$.

$$\overline{P} = \overline{L} \times \overline{C} \tag{17}$$

In order to calculate the drag on the cable segment, it is necessary to find the velocity components parallel and perpendicular to the segment. The drag coefficient for flow perpendicular to a cylinder is on the order of 100 times larger than the drag coefficient for flow parallel to it. Since in cable laying the component of cable velocity perpendicular to the cable is also much larger than the component parallel to the cable, the drag due to the velocity component parallel to the cable will be neglected.

The term $|\overline{V} \times \overline{L}|$ is the magnitude of the component of $\overline{V}$ which is perpendicular to $\overline{L}$, and $\overline{P}$ is the unit vector which is perpendicular to $\overline{L}$ and lies in the plane of $\overline{V}$ and $\overline{L}$. Therefore, the vector representation of the component of $\overline{V}$ which is perpendicular to $\overline{L}$ is:

$$\overline{V}_p = |\overline{V} \times \overline{L}|\ \overline{P} \tag{18}$$

When the cross products are expanded:

$$\overline{V}_p = V_{px}\,\overline{i} + V_{py}\,\overline{j} + V_{pz}\,\overline{k} \tag{19}$$

where:

$$V_{px} = L_y (V_x L_y - V_y L_x) - L_z (V_z L_x - V_x L_z) \tag{20}$$

$$V_{py} = L_z (V_y L_z - V_z L_y) - L_x (V_x L_y - V_y L_x) \tag{21}$$

$$V_{pz} = L_x (V_z L_x - V_x L_z) - L_y (V_y L_z - V_z L_y) \tag{22}$$

The drag on the cable segment is:

$$\overline{D} = D_x\,\overline{i} + D_y\,\overline{j} + D_z\,\overline{k} \tag{23}$$

where:

$$D_x = -\tfrac{1}{2}\rho C_D dL\ |\overline{V}_p|\ V_{px} \tag{24}$$

$$D_y = -\tfrac{1}{2}\rho C_D dL\ |\overline{V}_p|\ V_{py} \tag{25}$$

$$D_z = -\tfrac{1}{2}\rho C_D dL\ |\overline{V}_p|\ V_{pz} \tag{26}$$

where:

ρ = Mass density of seawater

C_D = Cable drag coefficient

d = Cable diameter

L = Cable segment length.

Since the cable segment is in equilibrium, the net force on it is zero, or:

(27) $$0 = \overline{T}_1 + \overline{T}_2 + \overline{D} + \overline{W}$$

where:

$\overline{T}_1$ = Cable tension at bottom of segment

$\overline{T}_2$ = Cable tension at top of segment

$\overline{D}$ = Drag on the cable segment

$\overline{W}$ = Net weight of cable segment

The vector equation (27) can be expanded into three scalar equations.

(28) $$0 = T_{1x} + T_{2x} + D_x$$

(29) $$0 = T_{1y} + T_{2y} + D_y$$

(30) $$0 = T_{1z} + T_{2z} + D_z + W_z$$

The only unknowns in equations 28-30 are the components of $\overline{T}_2$. However, the drag is a function of the average geometry of the cable segment which in turn is determined by the average tension in the cable segment. Since the drag equation is non-linear, an iterative solution is required.

ITERATIVE SOLUTION

The iterative solution is obtained as follows:

1. Assume $\overline{T}_2 = -\overline{T}_1$

2. Calculate the average tension in the cable segment, $\overline{T}_A = ½(\overline{T}_2 - \overline{T}_1)$.

3. Use $\overline{T}_A$ in equations 12-14 to calculate $\overline{L}$.

4. Use $\overline{T}_A$ in equations 5-7 to calculate $\overline{V}$.

5. Use equations 20-22 to calculate $\overline{V}_p$.

6. Use equations 24-26 to calculate $\overline{D}$.

7. Use equations 28-30 to calculate a new value for $\overline{T}_2$.

Steps 2-7 are iterated until the change in $\overline{T}_2$ is less than some predetermined amount.

CABLE GEOMETRY

Once a satisfactory value has been found for the tension at the top of the cable segment, the geometry of the segment is found from:

$$x_2 = x_1 + L \frac{T_{Ax}}{T_A} \tag{31}$$

$$y_2 = y_1 + L \frac{T_{Ay}}{T_A} \tag{32}$$

$$z_2 = z_1 + L \frac{T_{Az}}{T_A} \tag{33}$$

Now another cable segment can be added at the top of the previous one and the process continued until the required water depth is reached.

COMPUTER PROGRAM

A computer program was written which employs the calculation procedure derived above to calculate the geometry and tension of a cable being laid with non-zero bottom tension in a cross current. A listing of the programs, the required input variables, and a sample output are included in appendix A.

In the program, the iterative process which determines a new value for the tension at the top of a cable segment is stopped when the new $\overline{T}_2$ is less than 1 percent different from the previous value.

In general, the cable will be curved in the area near the bottom and will gradually approach the straight line geometry which it would have had if the bottom tension had been zero. An initial value is input for the cable segment length at the start of a computer run, but when the number of iterations required to get a solution for the new value of $\overline{T}_2$ becomes less than three, the length of the succeeding cable segments is doubled. Also, when the slope of the cable changes by less than 1 percent from one cable segment to the next, the cable is considered to be straight the rest of the way to the cable ship.

RESULTS

This paper is intended to be a presentation of a calculation method rather than the results of its application to a specific cable. However, an example will illustrate some of the cable laying concepts referred to in the Introduction and demonstrate the apparent validity of the computer program outputs.

Consider an example similar to one presented in reference 1. List 1 SD cable is being laid in 2000 fathoms of water at a ship speed of 5 knots. The cable characteristics are:

$$d = \text{Cable diameter} = 1.25 \text{ (inches)}$$

$$W_o = \text{Cable weight per foot in water} = .317 \text{ (lbs/ft)}$$

The drag coefficient normally used for a cylinder in cross flow is 1.2. However, according to reference 1 the results during cable laying experiments indicate that a drag coefficient of 3 should be used for cable laying calculations.

The transverse sinking speed, U_s, is determined from:

$$\text{Drag/ft} = \text{Weight/ft}$$

$$\tfrac{1}{2}\rho C_D d U_s^2 = W_o$$

$$U_s^2 = \frac{.317}{\tfrac{1}{2}\rho(3)\left(\frac{1.25}{12}\right)}$$

$$U_s = 1.0 \text{ (ft/sec)} = .6 \text{ (knots)}$$

The angle, α, at which the cable falls to the bottom as it is being laid at 5 knots is:

$$\alpha = \frac{U_s}{V_s} = \frac{.6}{5} = .12 \text{ (radians)}$$

$$\alpha = 6.9 \text{ (degrees)}$$

This shallow angle is typical of the cable angles which occur during cable laying. With this angle, the cable will touch bottom 16.5 nautical miles behind the cable ship.

If the cable is laid with zero bottom tension, the cable tension at the ship is equal to the cable weight per foot multiplied by the water depth.

$$T_s = W_o h = (.317)(12{,}000)$$

$$T_s = 3800 \text{ (lbs)}$$

It is important to realize that the cable geometry and tension remain virtually the same if the cable payout rate is just equal to the ship speed and no slack is laid or if cable payout rate is greater than ship speed and extra cable, or slack, is laid on the bottom.

According to reference 1, p 70, the presence of a uniform current perpendicular to the path of the cable ship will result in the cable being laid a distance, M, from the projection on the bottom of the path of the ship.

$$M = \frac{U_c}{U_s} h$$

where:

U_c = Velocity of current

U_s = Cable transverse sinking speed

h = Water depth

In our example, if the current velocity is equal to the transverse sinking speed of .6 knots, the cable would be laid 12,000 ft from where the ship passed.

The above example was input to the computer program with the cable tension at the ship equal to 8,000 lbs rather than 3800 lbs. In this case, the tension at the bottom is equal to 8000 - 3800 = 4200 lbs.

The sample computer output given in appendix A is the output for this example. Figure 8 shows the geometry of the lowest 10,000 feet of the approximately 100,000 feet of cable that is suspended in the water at any time. It is interesting to see that this large increase in cable tension produces a rather small change in cable geometry.

Figure 8 could also be interpreted as the geometry of a cable being laid in 1000 ft of water. In this case, for the straight line geometry, the tension at the bottom is zero and at the top is 317 lbs. For the curved geometry bottom tension is 4,200 lbs, and ship tension is 4,517 lbs.

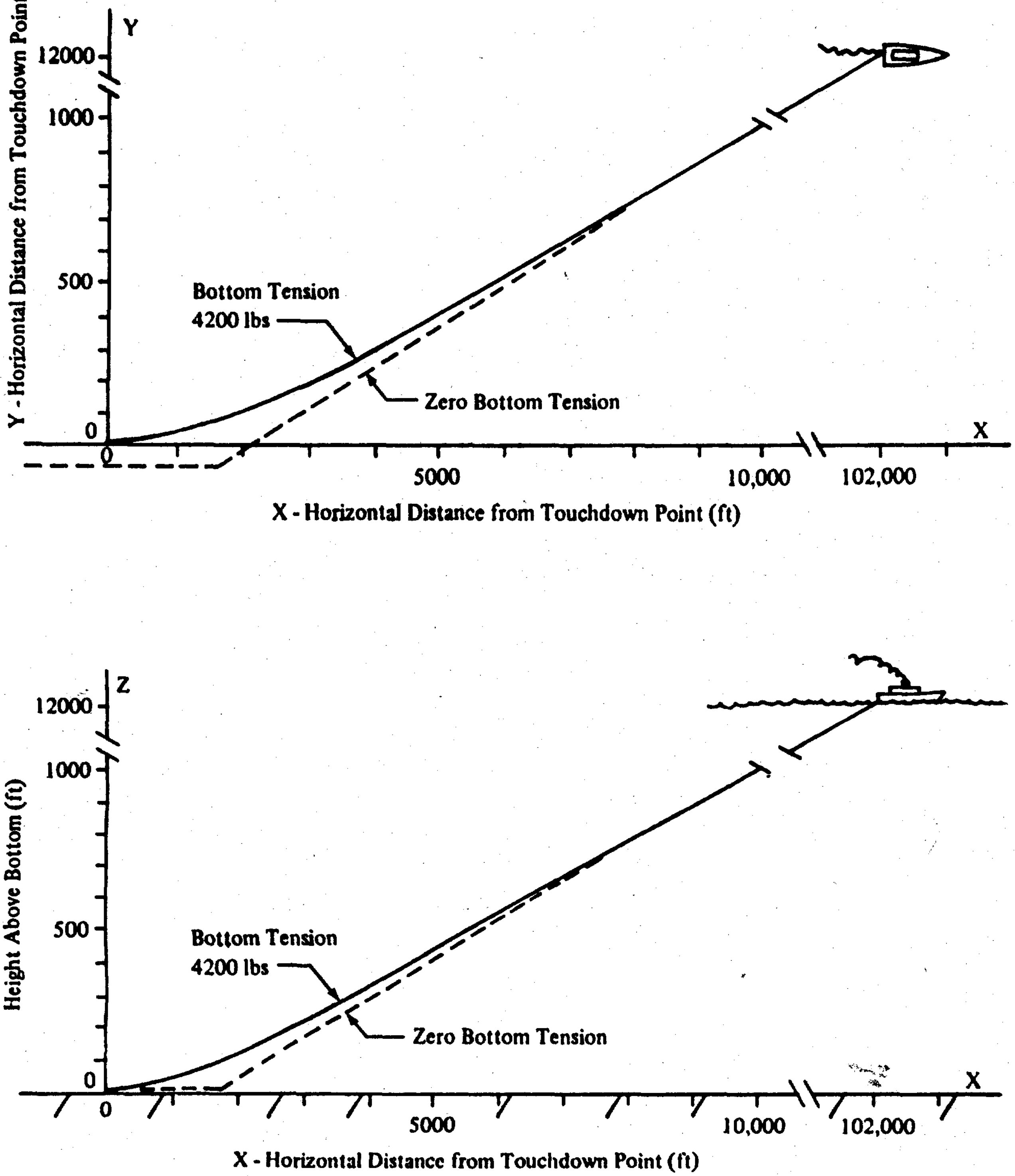

Figure 8. Cable geometry for the example problem.

APPENDIX A

COMPUTER PROGRAM

The input parameters required by the computer program are:

- L9 - Cable segment length at start of calculations (ft)
- W - Weight per foot of cable in water (lbs/ft)
- C1 - Cable drag constant, $\frac{1}{2}\rho C_D d$ (ft)
- V9 - Ship speed (ft/sec)
- V8 - Current speed (ft/sec)
- D9 - Water depth (ft)
- B1 - Cable tension at bottom (lbs)

The program listing is included as figure A-1. Figure A-2 is a typical output.

```
100 REM    (MBASIC VERSION)
110 REM CABLE LAYING GEOMETRY WITH A CROSS CURRENT
120 PRINT"INPUT L9,W,C1"
130 INPUT L9,W,C1
140 PRINT "INPUT V9,V8"
150 INPUT V9,V8
160 PRINT"INPUT D9,B1"
170 INPUT D9,B1
180 PRINT"INPUT PRINT OUT FLAG        +1  LPRINT          -1  NO LPRINT"
190 INPUT I
200 PRINT"     L9       W       C1      V9      V8      D9        B1"
210 A$ = " #### ##.##### ##.####  ##.##  ##.## #####.  ####.##"
220 PRINT USING A$;L9,W,C1,V9,V8,D9,B1
230 PRINT
240 PRINT
250 PRINT
260 PRINT"          X          Y          Z        TX        TY        TZ          T        N"
270 IF I<0 THEN 330
280 LPRINT STRING$ (10,10)
290 LPRINT"     L9       W       C1      V9      V8      D9        B1"
300 LPRINT USING A$:L9,W,C1,V9,V8,D9,B1
310 LPRINT STRING$(3,10)
320 LPRINT"         X          Y          Z        TX        TY        TZ          T        N"
330 B1=-B1
340 B2=0
350 B3=1E-04*B1
360 X=0
370 Y=0
380 Z=0
390 REM START A CABLE SEGMENT
400 B4=SQR(B1*B1+B2*B2+B3*B3)
410 N=0
420 T1=-B1
430 T2=-B2
440 T3=-B3
450 A1=(T1-B1)/2
460 A2=(T2-B2)/2
470 A3=(T3-B3)/2
480 A4=SQR(A1*A1+A2*A2+A3*A3)
490 N=N+1
500 L1=A1/A4
510 L2=A2/A4
520 L3=A3/A4
530 V1=V9*(1-A1/A4)
540 V2=V8-V9*A2/A4
550 V3=-V9*A3/A4
560 P1=L2*(V1*L2-V2*L1) - L3*(V3*L1-V1*L3)
570 P2=L3*(V2*L3-V3*L2) - L1*(V1*L2-V2*L1)
580 P3=L1*(V3*L1-V1*L3) - L2*(V2*L3-V3*L2)
590 P4=SQR(P1*P1+P2*P2+P3*P3)
600 D1=-C1*L9*P4*P1
610 D2=-C1*L9*P4*P2
620 D3=-C1*L9*P4*P3
```

Figure A-1. Computer program listing.

```
630 T5=-B1-D1
640 T6=-B2-D2
650 T7=-B3-D3+W*L9
660 REM CHECK CHANGE IN TENSION
670 IF ABS((T5-T1)/T1) > .01 THEN 710
680 IF ABS(T2/T3) < 1E-04 THEN 700
690 IF ABS((T6-T2)/T2) > .01 THEN 710
700 IF ABS((T7-T3)/T3) < .01 THEN 770
710 T1=T5
720 T2=T6
730 T3=T7
740 IF N<30 THEN 450
750 L9=L9*.75
760 GOTO 400
770 T1=T5
780 T2=T6
790 T3=T7
800 T4=SQR(T1*T1+T2*T2+T3*T3)
810 IF ABS((T1/T4+B1/B4)*B4/B1) > .01 THEN 860
820 IF ABS(B2/B3) < 1E-04 THEN 840
830 IF ABS((T2/T4+B2/B4)*B4/B2) > .01 THEN 860
840 IF ABS((T3/T4+B3/B4)*B4/B3) > .01 THEN 860
850 GOTO 1010
860 X=X+L9*A1/A4
870 Y=Y+L9*A2/A4
880 Z=Z+L9*A3/A4
890 B$ = " #######. ######.# #####.# #####.# #####.# #####.# #####.# #####."
900 PRINT USING B$;X,Y,Z,T1,T2,T3,T4,N
910 PRINT
920 IF I<0 THEN 950
930 LPRINT USING B$;X,Y,Z,T1,T2,T3,T4,N
940 LPRINT
950 IF N<3 THEN L9=L9*2
960 IF Z>= D9 THEN 1140
970 B1=-T5
980 B2=-T6
990 B3=-T7
1000 GOTO 400
1010 PRINT"CABLE STRAIGHT FROM HERE ON UP"
1020 IF I<0 THEN 1040
1030 LPRINT"CABLE STRAIGHT FROM HERE ON UP"
1040 V7=SQR(W/C1)
1050 V6=SQR(V9^2 + V8^2 + V7^2)
1060 T1=T1 + W*(D9-Z)*V9/V6
1070 T2=T2 + W*(D9-Z)*V8/V6
1080 T3=T3 + W*(D9-Z)*V7/V6
1090 T4=T4 + W*(D9-Z)
1100 X=X + (D9-Z)*V9/V7
1110 Y=Y + (D9-Z)*V8/V7
1120 Z=D9
1130 GOTO 890
1140 END
```

Figure A-1 (Continued). Computer program listing.

L9	W	C1	V9	V8	D9	B1
100	0.31700	0.3120	8.45	1.00	12000.	4200.00

X	Y	Z	TX	TY	TZ	T	N
100.	0.3	0.4	4199.9	29.4	31.2	4200.1	3.
200.	1.4	1.5	4199.7	55.6	60.2	4200.5	3.
300.	3.0	3.2	4199.4	79.3	87.8	4201.0	3.
400.	5.1	5.6	4199.0	100.8	114.2	4201.8	2.
600.	10.8	12.2	4198.4	139.0	163.5	4203.9	3.
799.	18.2	21.1	4198.0	172.5	208.6	4206.7	3.
999.	27.2	32.0	4197.8	202.5	249.6	4210.1	2.
1397.	48.9	59.0	4198.8	255.5	320.3	4218.7	3.
1795.	75.3	91.9	4201.7	301.0	375.4	4229.1	3.
2192.	105.5	129.2	4206.8	340.0	416.5	4241.0	3.
2589.	139.1	169.8	4213.9	372.9	446.1	4253.8	3.
2985.	175.5	212.8	4223.0	399.9	466.0	4267.5	2.
3776.	254.1	302.5	4244.9	440.9	491.0	4295.9	4.
4566.	338.3	394.6	4269.9	468.0	505.1	4325.1	3.
5356.	426.5	488.5	4297.4	485.0	511.3	4354.8	2.
6934.	607.8	677.2	4354.7	506.3	519.1	4414.7	4.
CABLE STRAIGHT FROM HERE ON UP							
101854.	11841.0	12000.0	7952.9	936.6	948.6	8063.8	2.

Figure A-2. Typical computer program output.